Algebra II 189-251B **Instructor: Dr. E. Goren.**

Assignment 11

To be submitted by April 13, 12:00

1. Find the Jordan canonical form of the matrix

$$A = \begin{pmatrix} 5 & 9 & -2 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 9 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}.$$

2. It is known that a differentiable function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ has a

- **maximum** at a point P if $\partial f/\partial x = \partial f/\partial y = 0$ at P and the 2×2 symmetric matrix
$$-\begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$$
is positive definite;
- **minimum** at a point P if $\partial f/\partial x = \partial f/\partial y = 0$ at P and the 2×2 symmetric matrix
$$\begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$$
is positive definite;
- **saddle point** at P if the 2×2 symmetric matrix
$$\begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$$
has one negative eigenvalue and one positive eigenvalue.

If P is either a maximum, minimum or saddle point, we call it a simple critical point. Determine the nature of the simple critical point of the following functions at the origin $(0, 0)$

$$f(x, y) = 2x^2 + 6xy + y^2, \quad f(x, y) = x \sin(x) - \cos(y) - xy.$$

(You may view the graphs and rotate them in Maple using
plot3d(2*xˆ 2+6*x*y+yˆ 2, x= -4..4, y = -4..4);
plot3d(x*sin(x) - cos(y) -x*y, x= -4..4, y = -4..4, numpoints=3000);).

Remark. This criterion can be generalized to functions $f : \mathbb{R}^n \longrightarrow \mathbb{R}$. If all the first partials vanish at a point P and the matrix of mixed derivatives is positive definite (resp. negative definite) at P, then the function has a minimum (resp. maximum) at P.

3. Find a formula for the general term of the sequence

$$0, 1, 4, 12, 32, 80, \ldots$$

$(a_{n+2} = 4a_{n+1} - 4a_n)$.

4. Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal, where

$$A = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}.$$