Algebra II 189-251B **Instructor: Dr. E. Goren.**

Assignment 10

To be submitted by April 6, 12:00

1. Let S and T be commuting linear maps from a vector space V to itself. In the last assignment we proved the following:

Let λ be an eigenvalue of T and let E_λ be the corresponding eigenspace. Prove that E_λ is S invariant. Conclude that if T is diagonalizable with eigenvalues $\lambda_1, \dots, \lambda_r$, and therefore

$$V = E_{\lambda_1} \oplus \cdots \oplus E_{\lambda_r},$$

we may decompose S as

$$S = S_1 \oplus \cdots \oplus S_r,$$

where $S_i : E_{\lambda_i} \longrightarrow E_{\lambda_i}$.

Assume that both S and T are diagonalizable. Prove now that there exists a basis of V in which both T and S are diagonal.

2. Let A be an $n \times n$ matrix over an algebraically closed field such that A^2 is diagonalizable. Prove that if A is a non-singular matrix then also A is a diagonalizable, and provide an example showing this condition is necessary.

3. Determine the possibilities for the Jordan canonical form of a matrix A with characteristic polynomial $\Delta_A(t) = (t-1)^6(t-2)^4(t-4)^5$ and minimal polynomial $m_A(t) = (t-1)^3(t-2)^2(t-4)$.

4. Prove that the matrix of complex numbers

$$A = \begin{pmatrix} 1 & 1 & 5 & 0 & 0 \\ 7 & 1 & 3 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

is diagonalizable. Note: you don't need to diagonalize it to prove it !!