ASSIGNMENT 3 - MATH 235, FALL 2009

Submit by 16:00, Monday, September 28 (use the designated mailbox in Burnside Hall, 10^{th} **floor).**

1. Find the quotient and remainder when a is divided by b:

   (1) $a = 302; b = 19$.
   (2) $a = -302; b = 19$.
   (3) $a = 0; b = 19$.
   (4) $a = 2000; b = 17$.
   (5) $a = 2001; b = 17$.
   (6) $a = 2008; b = 17$.

2. Prove that the square of any integer a is either of the form $4k$ or of the form $4k + 1$ for some integer k. (Hint: write a in the form $4q + r$, where $r = 0, 1, 2$ or 3.)

3. Prove of disprove: If $a|(b + c)$ then $a|b$ or $a|c$.

4. If $r \in \mathbb{Z}$ and r is a solution of $x^2 + ax + b$ (where $a, b \in \mathbb{Z}$) prove that $r|b$.

5. If $n \in \mathbb{Z}$, what are the possible values of
   (1) $(n, n + 2)$;
   (2) $(n, n + 6)$;
   (3) $(n, 2n + 1)$.

6. Suppose that $a > 1$ divides $n + 2$, divides $2n + 18$, and that a is odd. What's a?

7. Find the following gcd's. In each case also express (a, b) as $ua + vb$ for suitable integers $u, v \in \mathbb{Z}$.
   (1) $(56, 72)$.
   (2) $(24, 138)$.
   (3) $(143, 227)$.
   (4) $(314, 159)$.

8. If $a|c$ and $b|c$, must ab divide c? What if $(a, b) = 1$?

9. Give a bound for the number of steps in the Euclidean algorithm for finding (n, m) where $n \geq m > 0$ are integers.