ASSIGNMENT 3 - MATH 235, FALL 2009

Submit by 16:00, Monday, September 28 (use the designated mailbox in Burnside Hall, 10th floor).

1. Find the quotient and remainder when \(a\) is divided by \(b\):

   (1) \(a = 302; b = 19\).
   (2) \(a = -302; b = 19\).
   (3) \(a = 0; b = 19\).
   (4) \(a = 2000; b = 17\).
   (5) \(a = 2001; b = 17\).
   (6) \(a = 2008; b = 17\).

2. Prove that the square of any integer \(a\) is either of the form \(4k\) or of the form \(4k + 1\) for some integer \(k\). (Hint: write \(a\) in the form \(4q + r\), where \(r = 0, 1, 2\) or \(3\)).

3. Prove or disprove: If \(a| (b + c)\) then \(a|b\) or \(a|c\).

4. If \(r \in \mathbb{Z}\) and \(r\) is a solution of \(x^2 + ax + b\) (where \(a, b \in \mathbb{Z}\)) prove that \(r|b\).

5. If \(n \in \mathbb{Z}\), what are the possible values of
   (1) \((n, n + 2)\);
   (2) \((n, n + 6)\);
   (3) \((n, 2n + 1)\).

6. Suppose that \(a > 1\) divides \(n + 2\), divides \(2n + 18\), and that \(a\) is odd. What’s \(a\)?

7. Find the following gcd’s. In each case also express \((a, b)\) as \(ua + vb\) for suitable integers \(u, v \in \mathbb{Z}\).
   (1) \((56, 72)\).
   (2) \((24, 138)\).
   (3) \((143, 227)\).
   (4) \((314, 159)\).

8. If \(a|c\) and \(b|c\), must \(ab\) divide \(c\)? What if \((a, b) = 1\)?

9. Give a bound for the number of steps in the Euclidean algorithm for finding \((n, m)\) where \(n \geq m > 0\) are integers.