ASSIGNMENT 11 - MATH235, FALL 2009

DO NOT SUBMIT. THESE ARE SUGGESTED EXERCISES THAT MATCH THE MATERIAL COVERED AT PRESENT IN THE COURSE. SIMILAR QUESTIONS OFTEN APPEAR ON THE FINAL EXAM.

Solve questions (5), (12), (13) on pages 113-114.

Consider the group G of symmetries of the cube. Show that it has 48 elements; how many elements of order 2 does it have? Is there a symmetry of order 5 of the cube? What about order 6? Prove that the subgroup H of G consisting on symmetries that come from rotations only (no reflection), namely the orientation preserving symmetries, has 24 elements. By considering the faces of the cube, show that H and G can be realized as subgroups of S_6. Prove that G cannot be realized as a subgroup of S_5, hence it cannot be realized as a subgroup of S_4 (because we may view as S_4 as a subgroup of S_5). On the other hand, prove that H can be identified with S_4. (Hint: how can we extract from the cube a set of order 4 on which H acts?)

Find the number of roulettes and the number of necklaces with 12 stones, 2 red, 3 blue and 7 green.

Find the number of roulettes and the number of necklaces with 12 stones, 2 red, 4 blue and 6 green.

Find the number of roulettes and the number of necklaces with 14 stones, 2 red, 4 blue, 3 green and 5 black.