

QUIZ 3, MATH 235, FALL 2009

Date: November 17, 2009.

Time: 90 minutes.

Instructions: Notes, dictionaries or calculators are not NOT allowed. Write your answers clearly and with full details. Marks will be deducted for messy solutions. Write the solutions in the exam notebook.

- (1) Let \mathbb{F} be a field and $f(x) \in \mathbb{F}[x]$ a non-constant polynomial. Prove that $\mathbb{F}[x]/\langle f(x) \rangle$ is a field if and only if f is an irreducible polynomial. (You may assume it is a commutative ring.)
- (2) Construct a field \mathbb{F} with 8 elements. Prove that the polynomial $x^2 + x + 1$ does not have a solution in \mathbb{F} .
- (3) Construct a field \mathbb{F} with 121 elements. Prove that each of the polynomials $x^2 - 2$, $x^2 - 6$ and $x^2 + 2x + 2$ has two roots in \mathbb{F} and write them down explicitly.

Good luck!