Name ______________________, Student Number ________________

Version: a

QUIZ 2, MATH 235, FALL 2009

Date: November 3, 2009.
Time: 90 minutes.
Instructions: Answer all questions following the instructions. Notes, dictionaries or calculators are not NOT allowed. Write your answer clearly and with full details, where required. Marks will be deducted for ambiguous answers, or messy solutions. Use the notebook for answering part II and for scrap paper. Answer part I using the Answer Table on the first page.

Good luck!

PART I: Multiple choice questions. 60 points. Each question has a unique correct answer. Mark the correct answer in the table. Do not explain your answer, or show your work. Use the notebook for calculations. There is a penalty of -3 points for each wrong answer, beyond the first wrong answer (so x mistakes cost you $3(x-1)$ points, if $x \geq 1$). No penalty for leaving the question blank.

Question	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

(1) The congruence class of $((17)^{241} + 3^{16})^{66}$ modulo 5 is:
  (a) 0
  (b) 1
  (c) 2
  (d) 3
  (e) 4

(2) The number of rational roots of the polynomial $x^4 - 8 * x^3 - 29 * x^2 - 42 * x - 22$ is:
  (a) 0
  (b) 1
  (c) 2
  (d) 3
  (e) 4

(3) The number of real roots of the polynomial $x^4 - 8 * x^3 - 29 * x^2 - 42 * x - 22$ is:
  (a) 0
  (b) 1
  (c) 2
  (d) 3
  (e) 4

(4) The number of **distinct** roots in $\mathbb{Z}/5\mathbb{Z}$ of $x^4 - 8 * x^3 - 29 * x^2 - 42 * x - 22$ is:
  (a) 0
  (b) 1
  (c) 2
  (d) 3
  (e) 4

(5) The gcd of $x^3 - 9 * x^2 + 20 * x - 12$ and $x^3 - 5 * x^2 + 7 * x - 3$ over the field $\mathbb{Z}/5\mathbb{Z}$ is:
  (a) 1.
  (b) x.
  (c) $x - 1$.
  (d) $x^2 - 2 * x + 1$.
  (e) $x - 2$.
  (f) $x^2 - 3 * x + 2$.

(6) The gcd of $x^3 - 9 * x^2 + 20 * x - 12$ and $x^3 - 5 * x^2 + 7 * x - 3$ over $\mathbb{Q}$ is:
  (a) 1.
  (b) x.
  (c) $x - 1$.
  (d) $x^2 - 2 * x + 1$.
  (e) $x - 2$.
  (f) $x^2 - 3 * x + 2$.

(7) Which of the following sets is an ideal of the ring $Z[\sqrt{2}]$?
  (a) $\mathbb{Z}$.
  (b) $\{(3a + 2b) + (a + 3b)\sqrt{2} : a, b \in \mathbb{Z}\}$.
  (c) $\{b\sqrt{2} : b \in \mathbb{Z}\}$.
  (d) $\{2a + 3b\sqrt{2} : a, b \in \mathbb{Z}\}$.

(8) Which of the following is an ideal of $\mathbb{C}[x]$:
  (a) $\{f(x) = a_3x^3 + a_2x^2 + a_1x + a_0 : a_i \in \mathbb{C}\}$.
  (b) $\{f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0 : a_i \in \mathbb{R}, n \in \mathbb{N}\}$.
  (c) $\{f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0 : a_i \in \mathbb{C}, n \in \mathbb{N}, a_n + a_{n-1} + \cdots + a_1 + a_0 = 0\}$.
  (d) $\{f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0 : a_i \in \mathbb{C}, n \in \mathbb{N}, a_n + a_{n-1} = 0\}$.

(9) Let $f(x) = x^3 - 2 * x^2 - 3 * x + 6$, $g(x) = x^3 - 3 * x^2 + 3 * x - 2$ be rational polynomials. Apply the Euclidean algorithm to get polynomials $u(x), v(x)$ such that

$$\gcd(f(x), g(x)) = u(x)f(x) + v(x)g(x).$$

Let $m = \deg(u(x)) \cdot \deg(v(x))$. Then m is
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(10) For how many primes p does the polynomial $3x^2 + 3x + 11$ have precisely one root in $\mathbb{Z}/p\mathbb{Z}$? (That root will then be of multiplicity two).
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

Part II: proofs, 40 points. Write complete proofs for the following two statements. Write the proofs directly on the exam.

(1) Let $\mathbb{F}$ be a field. Any monic polynomial $f(x) \in \mathbb{F}[x]$ is a product of irreducible monic polynomials. (To clarify: you cannot use the Unique Factorization Theorem. The statement you are required to proof is (a slightly simplified version of) the existence part of that theorem.)
(2) Let $\mathbb{F}$ be a field. Any ideal in $\mathbb{F}[x]$ is a principal ideal.