ASSIGNMENT 5 - MATH235, FALL 2007

Submit by 16:00, Monday, October 15 (use the designated mailbox in Burnside Hall, 10^{th} **floor).**

1. To check if you had multiplied correctly two large numbers A and B, $A \times B = C$, you can make the following check: sum the digits of A; keep doing it repeatedly until you get a single digit number a. Do the same for B and C and get numbers b, c. If you have multiplied correctly, the sum of digits of ab is c. Prove that this is so. This is called in French "preuve par neuf".

Example: I have multiplied $A = 367542$ by $B = 687653$ and got $C = 252741358926$. To check (though this doesn't prove the mulitplication is correct) I do: $3+6+7+5+4+2 = 27, 2+7 = 9$ and $a = 9$. Also $6+8+7+6+5+3 = 35, 3+5 = 8$ and $b = 8$. $ab = 72$ and its sum of digits is 9. On the other hand $2+5+2+7+4+1+3+5+8+9+2+6 = 54, 5+4 = 9$. So it checks.

2.
(1) Solve that equation $x^2 + x = 0$ in $\mathbb{Z}/5\mathbb{Z}$.
(2) Solve that equation $x^2 + x = 0$ in $\mathbb{Z}/6\mathbb{Z}$.
(3) Solve that equation $x^2 + x = 0$ in $\mathbb{Z}/p\mathbb{Z}$, where p is prime.

3. Solve each of the following equations:
(1) $12x = 2$ in $\mathbb{Z}/19\mathbb{Z}$.
(2) $7x = 2$ in $\mathbb{Z}/24\mathbb{Z}$.
(3) $31x = 1$ in $\mathbb{Z}/50\mathbb{Z}$.
(4) $34x = 1$ in $\mathbb{Z}/97\mathbb{Z}$.
(5) $27x = 2$ in $\mathbb{Z}/40\mathbb{Z}$.
(6) $15x = 5$ in $\mathbb{Z}/63\mathbb{Z}$.

4.
(1) Let $p > 2$ be a prime. Prove that an equation of the form $ax^2 + bx + c$ (where $a, b, c \in \mathbb{F}_p, a \neq 0$) has a solution in $\mathbb{Z}/p\mathbb{Z}$ if and only if $b^2 - 4ac$ is a square in $\mathbb{Z}/p\mathbb{Z}$. If this is so, prove that the solutions are given by the familiar formula.
(2) Determine for which values of a the equation $x^2 + x + a$ has a solution in $\mathbb{Z}/7\mathbb{Z}$.

5. In each case, divide $f(x)$ by $g(x)$ with residue:
(1) $f(x) = 3x^4 - 2x^3 + 6x^2 - x + 2$, $g(x) = x^2 + x + 1$ in $\mathbb{Q}[x]$.
(2) $f(x) = x^4 - 7x + 1$, $g(x) = 2x^2 + 1$ in $\mathbb{Q}[x]$.
(3) $f(x) = 2x^4 + x^2 - x + 1$, $g(x) = 2x - 1$ in $\mathbb{Z}/5\mathbb{Z}[x]$.
(4) $f(x) = 4x^4 + 2x^3 + 6x^2 + 4x + 5$, $g(x) = 3x^2 + 2$ in $\mathbb{Z}/7\mathbb{Z}[x]$.