ASSIGNMENT 2 - MATH235, FALL 2007

Submit by 16:00, Monday, September 24 (use the designated mailbox in Burnside Hall, 10^{th} floor).

1. Consider $\mathbb{N} \times \mathbb{N}$ as a rectangular array:

$$\begin{array}{ccccc} (0,0) & (0,1) & (0,2) & (0,3) & \dots \\ (1,0) & (1,1) & (1,2) & (1,3) & \dots \\ (2,0) & (2,1) & (2,2) & (2,3) & \dots \\ (3,0) & (3,1) & (3,2) & (3,3) & \dots \\ \vdots & & & & \end{array}$$

Count them using diagonals as follows:

$$\begin{array}{cccccc} 0 & 1 & 3 & 6 & 10 & \dots \\ 2 & 4 & 7 & 11 & & \dots \\ 5 & 8 & 12 & & & \dots \\ 9 & 13 & & & & \dots \\ 14 & & & & & \dots \\ \vdots & & & & & \end{array}$$

This defines a function

$$f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$

where $f(m,n)$ is the number appearing in the (m,n) place. (For example, $f(0,0) = 0, f(3,1) = 13, f(2,2) = 12$.) Provide an explicit formula for f (it is what one calls "a polynomial function in the variables m, n". It may be a good idea to first find a formula for $f(0,n)$).

2. Prove that if $|A_1| = |A_2|$ and $|B_1| = |B_2|$ then $|A_1 \times B_1| = |A_2 \times B_2|$.

3. Prove that $|\mathbb{N}| = |\mathbb{Q}|$. (Hint: Show two inequalities; note that there is an easy injection $\mathbb{Q} \to \mathbb{Z} \times \mathbb{Z}$).

4. Prove or disprove: if $|A_1| = |A_2|$ and $|B_1| = |B_2|$ then $|A_1 \setminus B_1| = |A_2 \setminus B_2|$.

5. Prove that if a product $z_1 \cdot z_2$ of complex numbers is equal to zero then at least one of z_1, z_2 is zero.

6. Let f be the complex polynomial $f(x) = (3+i)x^2 + (-2-6i)x + 12$. Find a complex number z such that the equation $f(x) = z$ has a unique solution (use the formula for solving a quadratic equation).

7. Find the general form of a complex number z such that: (i) z^2 is a real number (i.e., $\mathrm{Im}(z^2) = 0$). (ii) z^2 is a purely imaginary number (i.e., $\mathrm{Re}(z^2) = 0$). Also, in each of these cases, plot the answer on the complex plane.