ASSIGNMENT 1 - MATH235, FALL 2007

Submit by 16:00, Monday, September 17 (use the designated mailbox in Burnside Hall, 10^{th} **floor).**

1. Calculate the following intersection and union of sets (provide short explanations, if not complete proofs).
   (1) Let $N \geq 0$ be a natural number. What is $\cup_{n=1}^{N}[-n,n]$? What is $\cap_{n=1}^{N}[-n,n]$?
   (2) What is $\cup_{n=1}^{\infty}[n,n+1]$? What is $\cup_{n=1}^{\infty}(n,n+2)$?
   (3) Let $A_n = \{x^n : x \in \mathbb{N}\}$. What is $\cup_{n=1}^{\infty}A_n$? What is $\cap_{n=1}^{\infty}A_n$?

2. Let A, B and C be sets. Prove or disprove:
   (1) $(A \setminus B) \setminus C = A \setminus (B \setminus C)$;
   (2) $(B \setminus A) \cup (A \setminus B) = (A \cup B) \setminus (A \cap B)$.

3. Prove that

$$A \setminus (\cap_{i \in I} B_i) = \cup_{i \in I}(A \setminus B_i).$$

4. Prove by induction that

$$1^3 + \cdots + n^3 = (1 + \cdots + n)^2.$$

You may use the formula for the r.h.s. we proved in class.

5. Let $A = \{1,2,3,4\}$ and $B = \{a,b,c\}$.
   (1) Write 4 different surjective functions from A to B.
   (2) Write 4 different injective functions from B to A.
   (3) How many functions are there from A to B?
   (4) How many surjective functions are there from A to B?
   (5) How many injective functions are there from A to B?
   (6) How many functions are there from B to A?
   (7) How many surjective functions are there from B to A?
   (8) How many injective functions are there from B to A?

6. Let $f : A \to B$ be a function.
   a) Prove that f is bijective if and only if there exists a function $g : B \to A$ such that $f \circ g = 1_B$ and $g \circ f = 1_A$.
   b) Prove or disprove: if there exists a function $g : B \to A$ such that $f \circ g = 1_B$ then f is bijective.

Notation: If a, b are real numbers we use the following notation:
$[a,b] = \{x \in \mathbb{R} | a \leq x \leq b\}$.
$[a,b) = \{x \in \mathbb{R} | a \leq x < b\}$.
$(a,b] = \{x \in \mathbb{R} | a < x \leq b\}$.
$(a,b) = \{x \in \mathbb{R} | a < x < b\}$.
We also use
$[a,\infty) = \{x \in \mathbb{R} | a \leq x\}$.
$(-\infty,b] = \{x \in \mathbb{R} | x \leq b\}$.
$(-\infty,\infty) = \mathbb{R}$.
If $A_1, A_2, A_3, \ldots$ are sets, we may write $\cup_{i=1}^{N}A_i$ for $A_1 \cup A_2 \cup \cdots \cup A_N$ and $\cup_{i=1}^{\infty}A_i$ for $\cup_{i \in \{1,2,3,\ldots\}}A_i$.