Algebra 4 (2003-04) – Assignment 7/8

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Submit by Monday, March 15, 12:00.

1) Construct fields $\mathbb{F}_4, \mathbb{F}_{16}$, of four and sixteen elements, respectively. For the field $\mathbb{F}_4$ write explicitly the addition and multiplication tables. Show that there are precisely two embeddings $\mathbb{F}_4 \hookrightarrow \mathbb{F}_{16}$ and write them down explicitly in terms of your construction of the fields.

2) Do Dummit and Foote, Ex. 8, §13.6, page 556.

3) Recall the definition of the Möbius function $\mu : \mathbb{N}^+ \longrightarrow \mathbb{Z}$. We let

$$\mu(n) = \begin{cases} 1 & n = 1 \\ 0 & n \text{ is divisible by a square} \\ (-1)^r & n = p_1 p_2 \dots p_r, r \text{ distinct primes.} \end{cases}$$

Prove the following:

(1) If $n > 1$ then $\sum_{d|n} \mu(d) = 0$. (The summation is over positive divisors of n, including 1 and n.)

(2) (Möbius inversion formula) Let $F(n) = \sum_{d|n} f(d)$. Then $f(n) = \sum_{d|n} \mu(d) F(n/d)$.