Algebra 4 (2003-04) – Assignment 6

Instructor: Dr. Eyal Goren

Submit by Monday, March 1, 12:00.

1) Prove that $r = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1$. Prove that one can construct a regular pentagon using straight-edge and compass and sketch the steps.

2) (Cf. Dummit and Foote, Ex. 5, 6, page 545).

(1) Let K be a finite extension of F. Prove that K is a splitting field over F if and only if every irreducible polynomial in $F[x]$ that has a root in K splits completely in $K[x]$.

(2) Let K_1, K_2 be finite extensions of F contained in the field K, and assume both are splitting fields over F. Prove that K_1K_2 and $K_1 \cap K_2$ are splitting fields over F.

Remark. (1) is the more difficult. You can do (the harder part of) (2) based on (1).