

Algebra 4 (2003-04) – Assignment 4

Instructor: Dr. Eyal Goren

Submit by Monday, February 9, 12:00 by mail-box on 10th floor.

1) (From Dummit and Foote, §12.3) Let $A \in M_2(\mathbb{Q})$ be a matrix satisfying $A^3 = I$, where I is the identity matrix. Assume $A \neq I$. Write A in rational canonical form and in Jordan canonical form viewed as a matrix over \mathbb{C} .

2)

- (1) (From Dummit and Foote, §13.1) Prove that $x^5 - ax - 1 \in \mathbb{Z}[x]$ is irreducible unless $a = 0, 2$ or -1 . The first two correspond to linear factors, the third corresponds to the factorization $(x^2 - x + 1)(x^3 + x^2 - 1)$.
- (2) Determine for which $b \in \mathbb{Z}$ the polynomial $x^2 + bx + 1$ is irreducible over \mathbb{Q} . In each of these cases determine whether the polynomial is also irreducible over $\mathbb{Q}(i)$.