Quiz 2,
Algebra 4, MATH 371
March 23, 2005

Time: 9:00 - 9:30.

1. Find the minimal polynomial of $\sqrt{3} + \sqrt{15}$ over $\mathbb{Q}$.
Hint: You may find it convenient to use the field $\mathbb{Q}(\sqrt{3}, \sqrt{5})$. You may use the fact that this field is Galois over $\mathbb{Q}$ and a description of its Galois group.

2. Let $\omega = e^{2\pi i/3}$, a primitive third root of 1. Sketch the argument that $\mathbb{Q}(\omega, \sqrt{3})/\mathbb{Q}$ is Galois and that
$$\text{Gal}(\mathbb{Q}(\omega, \sqrt{3})/\mathbb{Q}) \cong S_3.$$ 
For each subgroup of $S_3$ find the corresponding field under the Galois correspondence. Which fields are Galois over $\mathbb{Q}$?

**Bonus:** Is $\mathbb{Q}(i)$ a subfield of $\mathbb{Q}(\omega, \sqrt{3})$?

Good Luck!!