

**Quiz 2,**  
**Algebra 4, MATH 371**  
**March 23, 2005**

**Time:** 9:00 - 9:30.

1. Find the minimal polynomial of  $\sqrt{3} + \sqrt{15}$  over  $\mathbb{Q}$ .

Hint: You may find it convenient to use the field  $\mathbb{Q}(\sqrt{3}, \sqrt{5})$ . You may use the fact that this field is Galois over  $\mathbb{Q}$  and a description of its Galois group.

2. Let  $\omega = e^{2\pi i/3}$ , a primitive third root of 1. Sketch the argument that  $\mathbb{Q}(\omega, \sqrt[3]{2})/\mathbb{Q}$  is Galois and that

$$\text{Gal}(\mathbb{Q}(\omega, \sqrt[3]{2})/\mathbb{Q}) \cong S_3.$$

For each subgroup of  $S_3$  find the corresponding field under the Galois correspondence. Which fields are Galois over  $\mathbb{Q}$ ?

**Bonus:** Is  $\mathbb{Q}(i)$  a subfield of  $\mathbb{Q}(\omega, \sqrt[3]{2})$ ?

Good Luck!!