Quiz 1
Algebra 4–Math 371

Answer the following questions.

1. Let $R$ be an integral domain and $M$ an $R$-module. Let

$$Tors(M) = \{ m \in M : \exists r \in R, r \neq 0, rm = 0 \}.$$ 

You may assume it is an $R$-submodule of $M$.

Prove that

$$\text{rank}(M) = \text{rank}(M/Tors(M)).$$

2. Prove that two $3 \times 3$ matrices over a field $\mathbb{F}$ are similar over $\mathbb{F}$ if and only if they have the same minimal and characteristic polynomial.

**Bonus:** Give an explicit counterexample for this assertion for $4 \times 4$ matrices.