

Quiz 1
Algebra 4–Math 371
January 31, 2005.

Answer the following questions.

1. Let R be an integral domain and M an R -module. Let

$$\text{Tors}(M) = \{m \in M : \exists r \in R, r \neq 0, rm = 0\}.$$

You may assume it is an R -submodule of M .

Prove that

$$\text{rank}(M) = \text{rank}(M/\text{Tors}(M)).$$

2. Prove that two 3×3 matrices over a field \mathbb{F} are similar over \mathbb{F} if and only if they have the same minimal and characteristic polynomial.

Bonus: Give an explicit counterexample for this assertion for 4×4 matrices.