Algebra 4 (2004-05) – Assignment 8

Instructor: Dr. Eyal Goren

Submit by Monday, March 21, 24:00 by mail-box on 10th floor.

1. Let G be a finite group. Prove that there is a Galois extension of fields K/F such that $\text{Aut}(K/F) \cong G$.

(Hint: Show, for example, that S_n acts as automorphisms on the field $\mathbb{Q}(x_1, \ldots, x_n)$ – the field of fractions of the ring of polynomials in n variables $\mathbb{Q}[x_1, \ldots, x_n]$.)

2. Let p and ℓ be primes (equal or distinct). Let K be the splitting field over $\mathbb{Q}$ of the polynomial $x^p - \ell$. Prove that $K/\mathbb{Q}$ is Galois and that

$$\text{Gal}(K/\mathbb{Q}) \cong \mathbb{Z}/p\mathbb{Z} \rtimes (\mathbb{Z}/p\mathbb{Z})^\times.$$