Algebra 4 (2004-05) − Assignment 7

Instructor: Dr. Eyal Goren

Submit by Monday, March 14, 24:00 by mail-box on 10th floor.

Calculate the automorphism group $\text{Aut}(K/F)$ for the following pairs of fields:

(1) $F = \mathbb{Q}$ and K the splitting field of $x^3 + 3$.

(2) $F = \mathbb{Q}$ and K the splitting field of $(x^3 - 1)(x^2 + 3)$.

(3) $F = \mathbb{Q}$ and K the splitting field of $(x^3 - 1)(x^2 - 3)$.

(4) $F = \mathbb{C}$ and $K = \mathbb{C}(t)$. (Hint: If $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \text{GL}_2(\mathbb{C})$ then $t \mapsto \frac{at+b}{ct+d}$ defines an automorphism of $\mathbb{C}(t)/\mathbb{C}$.)

(5) $F = \mathbb{Q}$ and $K = \mathbb{Q}(\sqrt{2 + \sqrt{2}})$.

(6) $F = \mathbb{Q}(\sqrt{2})$ and $K = \mathbb{Q}(\sqrt{2 + \sqrt{2}})$.