

Algebra 4 (2004-05) – Assignment 5

Instructor: Dr. Eyal Goren

Submit by Monday, February 28, 24:00 by mail-box on 10th floor.

1) Prove that $r = 2 \cos(2\pi/5)$ satisfies the equation $x^2 + x - 1$. Prove that one can construct a regular pentagon using straight-edge and compass and sketch the steps.

2) (Cf. Dummit and Foote, Ex. 5, 6, page 545).

(1) Let K be a finite extension of F . Prove that K is a splitting field over F if and only if every irreducible polynomial in $F[x]$ that has a root in K splits completely in $K[x]$.

(2) Let K_1, K_2 be finite extensions of F contained in the field K , and assume both are splitting fields over F . Prove that K_1K_2 and $K_1 \cap K_2$ are splitting fields over F .

Remark. (1) is the more difficult. You can do (the harder part of) (2) based on (1).

3) Construct fields $\mathbb{F}_4, \mathbb{F}_{16}$, of four and sixteen elements, respectively. For the field \mathbb{F}_4 write explicitly the addition and multiplication tables. Show that there are precisely two embeddings $\mathbb{F}_4 \hookrightarrow \mathbb{F}_{16}$ and write them down explicitly in terms of your construction of the fields.