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Enumerating finite groups of given order.

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Let $n = \prod_{i=1}^{k} p_i^{g_i}$ be a positive integer with the p_i distinct primes. Let μ be the maximum of the g_i. The main result of this paper is that the number of groups of order n with preassigned isomorphism classes of Sylow subgroups is at most $n^{75\mu+16}$. Let $f(n)$ be the number of isomorphism classes of groups of order n. Combining this result with the Higman-Sims result on the number of p-groups of given order yields an interesting corollary: $f(n) \le n^{2/27+o(1)\mu^2}$ for $\mu \to \infty$.

The author first outlines a proof for the solvable case. There the idea is that the number of possibilities for the Fitting subgroup is limited and this limits the number of possibilities for the group. In the general case, the generalized Fitting subgroup is used. Now one has to use the classification of finite simple groups to derive various properties of simple groups.

This interesting paper contains related enumeration results and several conjectures as well. In particular, the author conjectures that almost all finite groups are nilpotent (in the sense that $f_1^*(n)/f^*(n) \to 1$ as $n \to \infty$, where $f^*(n)$ is the number of isomorphism classes of groups of order at most n and $f_1^*(n)$ is the number of isomorphism classes of nilpotent groups of order at most n).

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