Algebra 3 (2003-04) – Assignment 6

Instructor: Dr. Eyal Goren

Submit by Monday, October 20, 12:00 by mail-box on 10^{th} floor.

1) Write the conjugacy classes of S_4. For each conjugacy class choose a representative x and calculate its centralizer $C_{S_4}(x)$. Verify the class equation.

Do the same for A_4. Use the results to find the normal subgroups of A_4 and, in particular, deduce that A_4 does not contain a subgroup of order 6.

2) There is an obvious embedding of S_3 in S_6, the one in which S_3 acts on $\{1, 2, 3\} \subset \{1, 2, 3, 4, 5, 6\}$. This embedding is not transitive, that is, given $1 \leq i \leq j \leq 6$ we cannot always find an element of S_3 that takes i to j. Prove that there is a transitive embedding $S_3 \longrightarrow S_6$ (i.e., such that the image acts transitively on the 6 elements). Given such embedding, write the image of (12) and (123).

3) Prove that A_n is generated by 3 cycles for $n \geq 3$.

4) Is there an injective homomorphism $S_n \longrightarrow A_{n+1}$?

5) Let $\sigma \in S_n$. Prove that there is no odd permutation commuting with σ if and only if $p(\sigma)$ consists of distinct odd integers. (For example: (123) in S_3 satisfies this (because $p((123)) = 3$), but (123) does not satisfies that in S_5 (because $p((123)) = 1 + 1 + 3$).)

6) Write the conjugacy classes of A_6. Devise a direct proof that A_6 is simple.