Algebra 3 (2003-04) – Assignment 6

Instructor: Dr. Eyal Goren

Submit by Monday, October 20, 12:00 by mail-box on 10th floor.

1) Write the conjugacy classes of \( S_4 \). For each conjugacy class choose a representative \( x \) and calculate its centralizer \( C_{S_4}(x) \). Verify the class equation.

Do the same for \( A_4 \). Use the results to find the normal subgroups of \( A_4 \) and, in particular, deduce that \( A_4 \) does not contain a subgroup of order 6.

2) There is an obvious embedding of \( S_3 \) in \( S_6 \), the one in which \( S_3 \) acts on \( \{1, 2, 3\} \subset \{1, 2, 3, 4, 5, 6\} \). This embedding is not transitive, that is, given \( 1 \leq i \leq j \leq 6 \) we cannot always find an element of \( S_3 \) that takes \( i \) to \( j \). Prove that there is a transitive embedding \( S_3 \rightarrow S_6 \) (i.e., such that the image acts transitively on the 6 elements). Given such embedding, write the image of (12) and (123).

3) Prove that \( A_n \) is generated by 3 cycles for \( n \geq 3 \).

4) Is there an injective homomorphism \( S_n \rightarrow A_{n+1} \)?

5) Let \( \sigma \in S_n \). Prove that there is no odd permutation commuting with \( \sigma \) if and only if \( p(\sigma) \) consists of distinct odd integers. (For example: (123) in \( S_3 \) satisfies this (because \( p((123)) = 3 \)), but (123) does not satisfies that in \( S_5 \) (because \( p((123)) = 1 + 1 + 3 \)).)

6) Write the conjugacy classes of \( A_6 \). Devise a direct proof that \( A_6 \) is simple.