Algebra 3 (2003-04) – Assignment 3

Instructor: Dr. Eyal Goren

Submit by Monday, September 29, 12:00 by mail-box on 10^{th} floor.

1) Prove that a group of prime order is cyclic.

2) Prove that if $N < G$ and $[G : N] = 2$ then $N \triangleleft G$.

3) Find the subgroups of the quaternion group of order 8, Q. Prove that every subgroup of Q is normal. Suggestion: first find all cyclic subgroups of Q, then, by "pure thought" arguments, deduce that there are no other subgroups (except Q itself).

Note that this gives an example of a non-abelian group with the property that all its subgroups are normal.

In the following exercises use the fact that A_n is a simple group for $n \geq 5$.

4) Let $n \geq 5$. Prove that the only non-trivial normal subgroup of S_n is A_n.

5) Let $n \geq 5$. Prove that S'_n (the commutator subgroup of S_n) is equal to A_n.