Algebra 3 (2003-04) – Assignment 2

Instructor: Dr. Eyal Goren

Submit by Monday, September 22, 12:00 by mail-box on 10^{th} floor.

1) Find the center of the following groups:
(1) The dihedral group D_{2n} $(n \geq 3)$.
(2) The group $GL_n(\mathbb{F})$, where $\mathbb{F}$ is a field.

2) Calculate the commutator subgroup of D_{2n}; What's its order?

3) Recall the definition of Euler's φ function:

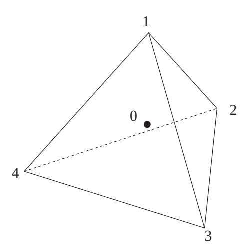
$$\varphi(n) = \sharp\{1 \leq a \leq n : (a, n) = 1\}, \qquad n = 1, 2, 3 \ldots .$$

(Thus, $\varphi(1) = 1,\ \varphi(2) = 1,\ \varphi(3) = 2,\ \varphi(4) = 2,\ \varphi(5) = 4,\ \varphi(6) = 2,\ \varphi(7) = 6,\ \varphi(8) = 4.$)
(1) Using the Chinese Remainder Theorem prove that if $(n, m) = 1$ then

$$\varphi(nm) = \varphi(n)\varphi(m).$$

(One says φ is a *multiplicative function*. The CRT says that if $(n, m) = 1$ then $\mathbb{Z}/n\mathbb{Z} \times \mathbb{Z}/m\mathbb{Z} \cong \mathbb{Z}/(nm)\mathbb{Z}$ as rings.)
(2) Prove that $\varphi(p^n) = p^n \left(1 - \frac{1}{p}\right)$.
(3) write a general formula for $\varphi(n)$.

4) Let G be the group of symmetries of a tetrahedron in $\mathbb{R}^3$. Prove that G can be identified with S_4.



5) Find all the subgroups of A_4 and display them as a graph with respect to inclusion.

NOTE: In this assignment you may make use of Lagrange's theorem: the order of a subgroup divides the order of a group.