November 5, 2004.

## Midterm Exam – MATH 370

Answer the following questions. The maximal grade possible is 110 out of 100.

Use of books or notes is not allowed. Use of calculators is allowed, but is not really necessary. Write your answers clearly and, preferably, in ink.

1. State and prove the class equation for a finite group G. (20 points)

2. Let G be a finite group and let p be a prime dividing its order. Prove that G has a p-Sylow subgroup. (25 points).

3. Find the number of necklaces with 9 beads, 2 red, 2 yellow, and 5 black. (25 points)

4. (i) Prove that a group of order 36 is solvable. Do not assume that the groups of smaller order coming up in your proof are solvable. Prove that as well! (20 points)

(ii) Show that there exist at least two non-isomorphic non-abelian groups of order 36. (20 points).

Good luck!!