Algebra 3 (2004-05) – Assignment 8

Instructor: Dr. Eyal Goren

Submit by Monday, November 15, 17:00 by mail-box on 10^{th} floor. Answer questions 1-4. Do not submit questions 5-6; they are additional questions for practice (and are not that hard in fact).

1). Prove that if G is a group of order $36, 40, 48, 54$ then G is not a simple group.

2). Find a composition series for each groups of order 8 and for the group S_4 having cyclic groups of prime order as quotients.

3). Find a normal series with abelian quotients for the subgroup $U = \{\begin{pmatrix} 1 & * & \dots & * \\ & 1 & \dots & * \\ & & \ddots & * \\ & & & 1 \end{pmatrix}\}$ of $\mathrm{GL}_n(\mathbb{F}_q)$.

4). Find $\mathrm{Inn}(Q), \mathrm{Aut}(Q)$ and $\mathrm{Out}(Q)$, where Q is the quaternion group of order 8..

5). Let p be a prime and let G be a finite group of order divisible by p. Let P be a p-Sylow subgroup of G and let N be a normal subgroup of G. Prove that $P \cap N$ is a p-Sylow subgroup of N. Give a counterexample if N is not normal.

6). Let p be a prime. Let G be a finite p-group. Let V be a finite dimensional vector space over a field with q-elements $\mathbb{F}_q$, $q = p^a$. Prove that there is a non-zero vector $v \in V$ such that every $g \in G$ fixes v.