Algebra 3 (2004-05) – Assignment 4

Instructor: Dr. Eyal Goren

Submit by Monday, October 11, 17:00 by mail-box on 10^{th} floor.

For the first 2 questions use suitable group actions.

1) Let G be a finite group. Let p be the minimal prime dividing the order of G and suppose that G has a subgroup K of index p. Prove that K is normal.

2) Let A be a proper subgroup of a finite group G. Prove that $G \neq \cup_{g \in G} gAg^{-1}$. Prove that this statement may fail for infinite groups (suggestion: Try $G = GL_2(\mathbb{C})$ for the second part).

3) Let S_3 act on $\mathbb{F}^3$, where $\mathbb{F}$ is a finite field, by permuting the coordinates. Find the number of orbits for this action. A size of an orbit is a divisor of 6 (why?). For each such divisor determine if there is an orbit of that size or not. (Either provide an example, or prove that none exists).

Consider the action of S_3 on the subspace given by $x_1 + x_2 + x_3 = 0$. How many orbits are there?

4) Find the number of necklaces with 12 stones – 2 red, 4 green, 3 blue and 3 yellow.