Algebra 3 (2004-05) – Assignment 3

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Submit by Monday, October 4, 17:00 by mail-box on 10^{th} floor.

1) If G, H are finite groups such that (|G|, |H|) = 1 then every group homomorphism $f : G \longrightarrow H$ is trivial $(f(G) = \{1\})$.

2) Find all possible homomorphisms $Q \longrightarrow S_3$. Is there an injective homomorphism $Q \longrightarrow S_4$? (As usual, Q is the quaternion group of order 8).

3) Prove that a group a non-abelian of order 6 is isomorphic to S_3 . Prove that every abelian group of order 6 is isomorphic to $\mathbb{Z}/6\mathbb{Z}$.

Here are some hints: start by showing that every group G of order 6 must have an element x of order 2 and an element y of order 3. This in fact follows from some general theorems but I want you to argue directly using only what we covered in class. (A typical problem here is why can't all the elements different from 1 have order 3. If this is the case, show that there are two cyclic groups K_1, K_2 of G of order 3 such that $K_1 \cap K_2 = \{1\}$. Calculate $|K_1K_2|$.)

Having shown that, if G is abelian show it implies the existence of an element of order 6. In the nonabelian case show that we must have $xyx^{-1} = y^2$ and that every element in G is of the form x^ay^b , a = 0, 1, b = 0, 1, 2. Show that the map $x \mapsto (1 \ 2), y \mapsto (1 \ 2 \ 3)$ extends to an isomorphism.