1) Find the lattice of subgroups of the alternating group $A_4$. Determine which subgroups are normal. Do the same for the quaternion group $Q$ of order 8. (By a lattice, I mean that you also indicate which subgroup is contained in which. Here is what happens for $S_3$ and $\mathbb{Z}/8\mathbb{Z}$

Also note that the minimal subgroups are always cyclic. The ones above them are generated by 2 elements, etc. This tells you how to go about finding all the subgroups of a given group. )

2) In this exercise you are required to calculate the commutator subgroup and center of some groups.
   (1) Find the center of the following groups: $D_{2n}, GL_n(\mathbb{F})$, where $\mathbb{F}$ is any field.
   (2) Find the commutator subgroup of $D_{2n}$.
   (3) Prove that the commutator subgroup of $GL_n(\mathbb{F})$ is contained in $SL_n(\mathbb{F})$, $\mathbb{F}$ a field. (In fact equality holds. Optional: prove that for $n = 2$.)

3) Prove that if $N < G$ and $[G : N] = 2$ then $N\triangleleft G$.

4) Prove that a group of prime order is cyclic.