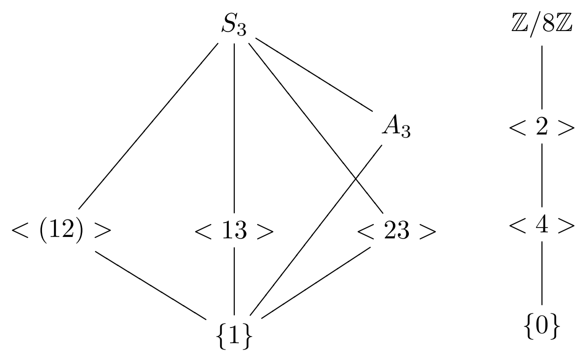
Algebra 3 (2004-05) – Assignment 2

Instructor: Dr. Eyal Goren

Submit by Monday, September 27, 12:00 by mail-box on 10^{th} **floor.**

1) Find the lattice of subgroups of the alternating group A_4. Determine which subgroups are normal. Do the same for the quaternion group Q of order 8.
(By a lattice, I mean that you also indicate which subgroup is contained in which. Here is what happens for S_3 and $\mathbb{Z}/8\mathbb{Z}$



Also note that the minimal subgroups are always cyclic. The ones above them are generated by 2 elements, etc. This tells you how to go about finding all the subgroups of a given group.)

2) In this exercise you are required to calculate the commutator subgroup and center of some groups.

(1) Find the center of the following groups: $D_{2n}, GL_n(\mathbb{F})$, where $\mathbb{F}$ is any field.
(2) Find the commutator subgroup of D_{2n}.
(3) Prove that the commutator subgroup of $\mathrm{GL}_n(\mathbb{F})$ is contained in $\mathrm{SL}_n(\mathbb{F})$, $\mathbb{F}$ a field. (In fact equality holds. Optional: prove that for $n = 2$.)

3) Prove that if $N < G$ and $[G : N] = 2$ then $N \lhd G$.

4) Prove that a group of prime order is cyclic.