Algebra 3 (2004-05) – Assignment 1

Instructor: Dr. Eyal Goren

Submit by Monday, September 20, 12:00 by mail-box on 10^{th} floor.

1) Let G be a finite group such that for every $g \in G$ we have $g^2 = e$.

(1) Prove that G is an abelian group.
(2) Prove that G has 2^n elements.

2) Write down all the elements of $\mathrm{GL}_2(\mathbb{F}_2)$. Consider the action of this group on the set of non-zero vectors in $\mathbb{F}_2^2$ (the two dimensional vector space over $\mathbb{F}_2$). Show that this allows one to identify the group $\mathrm{GL}_2(\mathbb{F}_2)$ with the symmetric group S_3.

3) Let D_{2n} be the dihedral group with $2n$ elements. It is generated by x, y, satisfying $x^n = y^2 = xyxy = 1$. Prove (algebraically) that every element not in the subgroup $\langle x \rangle$ is a reflection and find (geometrically) the line through which it is a reflection.

4) Show that the transpositions $(12), (23), \dots, (n-1\ n)$ generate S_n. Show that (12) and $(1\,2\,3 \dots n)$ generate S_n.