ASSIGNMENT 8 - ALGEBRAIC GEOMETRY 189-706 A

EYAL Z. GOREN   NOVEMBER 1, 1999

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1. Solve exercise 6.6 in Hartshorne, p. 46. (Hartshorne's notation PGL(1) is highly non-standard. This is the group we denoted $\mathrm{PGL}_2(k)$ or $\mathrm{PGL}(2, k)$. The one obtained from 2×2 matrices.)

2. A) Find the singular locus of

$$y^2 = x_1 \cdots x_n$$

in $\mathbb{A}^n_{y,x_1,\dots,x_n}$.
   B) Is it a normal variety? (Hint: some of the computations are not unlike computing the ring of integers in a quadratic extension of $\mathbb{Q}$. You might want to check first the cases $n = 1, 2$).

3. Give a (sufficient) criterion for $y^2 = f(x)$ in $\mathbb{A}^n_{y,x_1,\dots,x_n}$ to be: (i) Non-singular; (ii) Normal.

4. Find the normalization of

$$y^2 = x_1 x_2^2.$$

Study the situation in detail !