1. Prove that $R$ is noetherian implies that $R_p$ is noetherian for every prime ideal $p \lhd R$.

2. Let $R$ be a domain. Prove that $R$ has dimension one if and only if $R_p$ has dimension one for every prime ideal $p \lhd R$.

3. Fix a rational prime $p$. For an integer $a$ define $v(a)$ to be the largest power of $p$ dividing $a$. For a rational number $m = a/b$ let $v(m) = v(a) - v(b)$. Show that $v$ is a valuation on $\mathbb{Q}$.

4. Prove that a principal local domain $(R, m)$ is a DVR. Use that a PID is a UFD. Note that $m = (\pi)$ for $\pi$ irreducible. Given $r \in R$ write $r = \pi^{v(r)} \cdot r'$, with $(r', \pi) = (1)$, and some integer $v(r)$.

5. Let $A \subset B$ be two DVRs with the same quotient field. Show that $A = B$.

6. A) Consider the curve $\mathcal{C} : y^2 = x^2(x + 1)$ and the point $(0, 0)$ on it. Show that the local ring of $(0, 0)$ is not a DVR. Find the normalization $\mathcal{D}$ of $\mathcal{C}$.

   B) Show that the pre-image of $(0, 0)$ consists of two points. Say, $s$ and $t$. Let $v_s$ and $v_t$ be the corresponding valuations. Give a function on $\mathcal{D}$ with different valuation at $s$ and $t$. How does this relate to A)?