ASSIGNMENT 7 - ALGEBRAIC GEOMETRY 189-706 A

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To be submitted Wednesday, November 3.

1. Prove that R is noetherian implies that $R_{\mathfrak{p}}$ is noetherian for every prime ideal $p \lhd R$.

2. Let R be a domain. Prove that R has dimension one if and only if $R_{\mathfrak{p}}$ has dimension one for every prime ideal $p \lhd R$.

3. Fix a rational prime p. For an integer a define $v(a)$ to be the largest power of p dividing a. For a rational number $m = a/b$ let $v(m) = v(a) - v(b)$. Show that v is a valuation on $\mathbb{Q}$.

4. Prove that a principal local domain $(R, \mathfrak{m})$ is a DVR. Use that a PID is a UFD. Note that $\mathfrak{m} = (\pi)$ for π irreducible. Given $r \in R$ write $r = \pi^{v(r)} \cdot r'$, with $(r', \pi) = (1)$, and some integer $v(r)$.

5. Let $A \subset B$ be two DVRs with the same quotient field. Show that $A = B$.

6. A) Consider the curve $\mathcal{C} : y^2 = x^2(x+1)$ and the point $(0,0)$ on it. Show that the local ring of $(0,0)$ is not a DVR. Find the normalizaion $\mathcal{D}$ of $\mathcal{C}$.

B) Show that the pre-image of $(0,0)$ consists of two points. Say, s and t. Let v_s and v_t be the corresponding valuations. Give a function on $\mathcal{D}$ with different valuation at s and t. How does this relate to A)?