ASSIGNMENT 5 - ALGEBRAIC GEOMETRY 189-706 A

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To be submitted Monday, October 11.

1. Prove, using the projection from a point, that there are infinitely many solutions to the equation
$$x^2 + y^2 = z^2$$
in rational numbers. Can you actually write them all?

2. Blow up the surface $xy^2 = z^2$ at 0. Find the ideal defining the strict transform and the singular points of the blow-up (if any).

3. Let Y be a subvariety of $\mathbb{A}^n$ passing through zero. Prove that $\widetilde{Y} \cap \phi^{-1}(0)$ is not empty. (I.e., that the strict transform intersect the special fibre).

4.* Prove that the blow-up of the cone is locally, but not globally isomorphic to $\mathbb{A}^1 \times \mathbb{P}^1$.

5. Show that $\mathbb{P}^1 \times \mathbb{P}^1$ is birational to $\mathbb{P}^2$ but not isomorphic to it.

6. Solve question 4.3 in Hartshorne.

7. Solve question 4.7 in Hartshorne.

Remark: In some of the questions, e.g. those involving products, you may use any fact given in any exercise in Hartshorne preceding those you are solving in this assignment.
$\star$ As a general rule: exercises marked with $*$ can be submitted anytime during the semester.