ASSIGNMENT 4 - ALGEBRAIC GEOMETRY 189-706 A

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To be submitted Monday, October 27.

Localization. Let R be a commutative ring. Let M be an R-module and U a multiplicative set in R. Prove:

1. The kernel of the map $\phi : M \longrightarrow M[U^{-1}]$ is $\{m : \exists u \in U, um = 0\}$.
2. Prove that there is a bijection between ideals of $R[U^{-1}]$ and ideals of R disjoint with U that is given by

$$I \mapsto \phi^{-1}(I), \quad J \mapsto \phi(J)R[U^{-1}].$$

Show also that it takes a prime ideal to a prime ideal.

3. Prove that $\mathfrak{p} \triangleleft R$ is a unique maximal ideal if and only if $R^{\times} = R \setminus \mathfrak{p}$.

4. Study the proofs of Proposition 3.3, Theorem 3.4 and Proposition 3.5 in Hartshorne pp. 18-19. Note that Theorem 3.4 strengthens slightly what we did in class. (Nothing to submit!!)

5. Prove that the closure of a q. affine variety in projective space (via the map φ_o given in class) is a projective variety of the same dimension. (Use some recent tools....). Give an example of an irreducible noetherian topological space X with a dense open set U such that $\dim(U) < \dim(X)$.
6. Do question 3.15 in Hartshorne p.22.
7.* Discuss the following fugue in three voices:

- A famous theorem of Chow asserts the following:

Let Z be a compact analytic sub-manifold of $\mathbb{P}^n(\mathbb{C})$. Then Z is an algebraic variety.
(This is very hard to prove. You're not expected to prove it but to contemplate it... If you heard about Riemann surfaces and the canonical mapping you might want to examine the consequences!).

- Convince me that the set $\{(t, sin(t)) : t \in \mathbb{C}\}$ is Zariski dense in $\mathbb{A}^2(\mathbb{C})$.
- Recall that $sin(t)$ has an essential singularity at infinity.

$\star$ As a general rule: exercises marked with $*$, as 1.11, or 2.17, can be submitted anytime during the semester.