MCGILL UNIVERSITY - DEPARTMENT OF MATHEMATICS

TOPICS IN GEOMETRY AND TOPOLOGY I & II – ALGEBRAIC GEOMETRY 189-706A, 189-707B

Lecturer: Eyal Goren. (Tel: 3815, Office 1108, email: goren@math.mcgill.ca)

General description: Two consecutive courses in algebraic geometry will be given in 1999-2000. Students may take either one or both, but the second semester assumes the first as a pre-requisite. This means that taking only the second semester is possible only if the student took a course similar to the one given in the first semester or have received my approval for joining only for the second.

We will follow Hartshorne (see below) very closely, with some material (normalization, tangent cone) taken from Mumford. There will be a great emphasis on assignments which will also constitute a large share of the grade.

Algebraic geometry is the study of varieties (curves, surfaces, etc...) defined, at least locally, as the set of solutions of a system of polynomials. Much effort went into defining such objects in an intrinsic way, "freeing" them from the description in terms of polynomials, and into properly articulating the notion of a continuous family of varieties. In that, one tries to put on equal footing the idea that a family of varieties $(V_t : t \in C)$ parameterized by a curve C, and a *single* variety defined by polynomials with integer coefficients are similar, since the later defines also a family of varieties obtained by reducing the equations modulo every prime. This leads to the notion of schemes.

After establishing some honest geometric back-ground by discussing varieties in the naive sense (solutions of polynomials over an algebraically closed field), we'll shift to the language of schemes. One of the high lights will be the study of intersection theory and its many applications.

Prerequisites: The course is a graduate course and is open to advanced undergraduates. The students are supposed to know basic algebra (Rings, modules) and elementary topology (very basic notions like open, closed, boundary). They are expected to catch-up on commutative algebra during the course, as necessary. Precise references would be given.

Text books: The main text book for this course is

Hartshorne, Robin: Algebraic geometry. Graduate Texts in Mathematics, No. 52. Springer-Verlag, New York-Heidelberg, 1977. xvi+496 pp. ISBN: 0-387-90244-9

This is the best text-book around and is highly recommended as an investment. We will use the following book for catching up on algebra

Eisenbud, David: Commutative algebra. With a view toward algebraic geometry. Graduate Texts in Mathematics, 150. Springer-Verlag, New York, 1995. xvi+785 pp.
ISBN: 0-387-94268-8; 0-387-94269-6

Some sections will be taken from

Mumford, David: The red book of varieties and schemes. Lecture Notes in Mathematics, 1358. Springer-Verlag, Berlin-New York, 1988. vi+309 pp. ISBN: 3-540-50497-4

Syllabus: Affine and projective varieties. Morphisms. Rational maps. Non-singular varieties. Intersections in a projective space.

Sheaves. Schemes and their basic properties. Separated and proper morphisms. Coherent, quasi-coherent sheaves and differentials. Divisors and Projective morphisms.

Cohomology and Cech cohomology. Cohomology of affine and projective spaces. Serre's duality. The canonical divisor and Riemann-Roch theorem for curves. Flat, étale and smooth morphisms. The étale site and the fundamental group.

The Chow ring. Characteristic classes. K groups and the general Grothendieck-Hirzebruch-Riemann-Roch theorem.

This announcement can be also down-loaded from http://www.math.mcgill.ca/ goren/

Eyal Goren. August 6, 1999