MATH 765 ASSIGNMENT 3

DUE WEDNESDAY NOEVEMBER 6

1. Show that $\mathrm{Lip}(1, L^p(\mathbb{T}^n)) = W^{1,p}(\mathbb{T}^n)$ for $1 < p \leq \infty$.
2. Prove that $[L^p(\mathbb{T}^n), W^{1,p}(\mathbb{T}^n)]_{\alpha,q} = B^{\alpha}_{p,q}(\mathbb{T}^n)$ for $0 < \alpha < 1$ and $1 \leq p, q \leq \infty$.
3. Show that $B^{\alpha}_{p_1,q}(\mathbb{T}^n) \hookrightarrow B^{\alpha}_{p_2,q}(\mathbb{T}^n)$ if and only if $p_1 \geq p_2$ (assume $0 < \alpha < 1$).
4. Prove that $[L^p(\mathbb{T}), W^{2,p}(\mathbb{T})]_{\theta,q} = B^{2\theta}_{p,q}(\mathbb{T})$ for $0 < \theta < 1$ and $1 \leq p, q \leq \infty$.
5. Show that $B^{1}_{p,q}(\mathbb{T}) \neq W^{1,r}(\mathbb{T})$ unless $p = q = r = 2$.
6. Let $\Omega \subset \mathbb{R}^2$ be a bounded polygonal domain, and let S^d_P be the standard Lagrange finite element spaces, where $d > 0$ is the local polynomial order and P is a conforming partition of Ω, with $\#P > 1$. For which values of the indices p, q and α do we have the inclusion $S^d_P \subset B^{\alpha}_{p,q}(\Omega)$?

Date: Fall 2013.