MATH 595/597 ASSIGNMENT 3

DUE FRIDAY NOVEMBER 7

1. By way of a counterexample, show that $H^s(\mathbb{T}^n)$ with $s = \frac{n}{2}$ is not closed under pointwise multiplication of functions.
2. Prove that the Navier-Stokes initial value problem
$$\partial_t u = \Delta u - \mathbb{P}\mathrm{div}(u \otimes u),$$
in $\mathbb{T}^4$ or $\mathbb{T}^5$, with smooth, divergence-free initial data, has a global smooth solution if the H^3-norm of the initial data is small.
3. Prove global solvability of the magnetohydrodynamics system
$$\begin{aligned} \partial_t u &= \Delta u - u \cdot \nabla u - \nabla p + h \cdot \nabla h, \\ \partial_t h &= \Delta h - u \cdot \nabla h + h \cdot \nabla u, \\ \operatorname{div} u &= \operatorname{div} h = 0, \end{aligned}$$
in $\mathbb{T}^2$. Here u (velocity) and h (magnetic field) are time-dependent vector fields, and the initial data for both of them are assumed to be smooth and divergence-free. Note that one needs to address the issue that we have 5 unknowns and 6 equations (In other words, there is no "pressure term" in the evolution equation for h).

Date: October 28, 2014.