

## MATH 580 ASSIGNMENT 6

DUE THURSDAY DECEMBER 1

1. Prove a local well-posedness result together with a blow-up criterion for the  $n$ -dimensional Burgers' equation

$$\partial_t u + u \cdot \nabla u = \Delta u,$$

where  $u : \mathbb{R}^n \times (0, T) \rightarrow \mathbb{R}^n$ .

2. Show that in  $\mathbb{R}^3$ , the wave propagators form a one parameter group of linear operators.
3. Consider the Cauchy problem for the wave equation  $\partial_t^2 u - \Delta u = 0$  in  $\mathbb{R}^n$ . Suppose that  $u : \mathbb{R}^n \times \mathbb{R}^1 \rightarrow \mathbb{R}$  is its solution, and that the initial data vanish outside  $\Omega \subset \mathbb{R}^n$ . Show that  $u(x, t) = 0$  if  $|t| < \inf_{y \in \Omega} |x - y|$ .
4. (*Bonus problem*) Prove that the  $n$ -dimensional Burgers' equation is globally well-posed.