MATH 578 MIDTERM PRACTICE QUESTIONS

FALL 2009

1. Let $A \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix, and let $A = LL^T$ be its Cholesky factorization. Show that if A is banded, then so is L.
2. Describe how Gaussian elimination with complete pivoting succeeds when A is singular.
3. Exercises 5 and 12 in §3.15 of the textbook.
4. Is the matrix
$$A = \begin{bmatrix} 2 & -1 & & \\ -1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{bmatrix} \in \mathbb{R}^{n\times n},$$
positive definite?
5. Let $A \in \mathbb{R}^{n\times n}$ be a symmetric, positive definite matrix, and let $x_0 = 0, x_1, x_2, \ldots, x_n$, denote the iterates of the CG method applied to solve $Ax = b$. Show that $\|x_{i+1}\|_2 \geq \|x_i\|_2$ for $i = 0, \ldots, n-1$.
6. What is the motivation behind the notion of numerical stability?
7. Let A be a symmetric matrix, and let $\lambda_1 \leq \ldots \leq \lambda_n$ be its eigenvalues. When A is positive definite (i.e., $\lambda_1 > 0$), how can one interpret the linear system $Ax = b$ as a quadratic minimization problem? What becomes of the minimization problem if A is singular and positive semi-definite (i.e, $\lambda_1 = 0$)? What if A is regular but indefinite (i.e., $\lambda_i < 0 < \lambda_{i+1}$ for some i)? Discuss the applicability of the steepest descent method for each case.
8. Discuss the main ideas involved in the CG method.