MATH 319 ASSIGNMENT 5

DUE FRIDAY APRIL 15

Part A

Solve the following problems from the textbook.

- §9.1: 2, 7, 8, 10, 13, 14 (In 14, you can use a calculator to get approximate results.)
- §9.2: 1, 2
- §9.3: 7, 8

Part B

1. For each of the following situations in the xy-plane, compute the monopole, dipole, and quadrupole moments, and write down the multipole expansion of the electrostatic potential up to (and including) the quadrupole term. Normalize the constant so that the potential of a unit charge at the origin would be $\log \frac{1}{r}$, where r is the radial coordinate.
   (a) The rectangle $R = \{(x, y) : -1 \leq x \leq 1, -h \leq y \leq h\}$ with total charge 1, and uniform charge density. Here $h > 0$ is a given constant.
   (b) The same as in (a), but now the half $R \cap \{x > 0\}$ of the rectangle has total charge $+1$ with uniform density, and the other half has total charge -1 with uniform density.
2. Let $u(x, y, t)$ be a smooth function satisfying

$$\begin{cases} u_{tt} = u_{xx} + u_{yy} & \text{in } \Omega, \quad \text{for } t > 0, \\ u = 0 & \text{on } \partial\Omega, \quad \text{for } t > 0, \\ u(x, y, 0) = f(x, y) & \text{for } (x, y) \in \Omega, \\ u_t(x, y, 0) = g(x, y) & \text{for } (x, y) \in \Omega, \end{cases}$$

   where $\Omega \subset \mathbb{R}^2$ is a bounded region in the plane, and f and g are given initial data. In other words, u is a solution of the Dirichlet initial-boundary value problem for the wave equation in the domain Ω. By using the energy method, show that u is the unique solution of this problem, i.e., that there are no other solutions. *Hint*: Suppose that $v(x, y, t)$ is another solution of the problem (with the same initial and boundary data), and consider the energy

$$E(t) = \int_\Omega (w_t^2 + w_x^2 + w_y^2)$$

   for difference $w = u - v$. Compute the time derivative $E'(t)$. Apply the divergence theorem to the vector field $F = w_t \operatorname{grad} w$.

Date: Winter 2016.