

MATH 319 ASSIGNMENT 2

DUE WEDNESDAY FEBRUARY 10

PART A

From §5.2 of the textbook, solve the following problems: 1(ace), 5, 8.

PART B

1. Consider the wave equation

$$u_{tt} = c^2 u_{xx},$$

on the whole line, with the initial data

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x).$$

Suppose that f is fixed, and that we have some freedom in choosing g . For instance, we know that if $g \equiv 0$ then the solution $u(x, t)$ is simply the initial profile f split in two halves, travelling in opposite directions with speed c . However, we do not want f to be split. We want the solution $u(x, t)$ to be equal to the initial profile f travelling to the right with speed c . What should g be if this is the case? What would the choice be if we want the solution $u(x, t)$ to be equal to the initial profile f travelling to the *left* with speed c ? In each case, provide a concrete example of f and g .

2. Solve the problem

$$\begin{cases} u_{tt} = c^2 u_{xx}, & 0 \leq x \leq L, \quad -\infty < t < \infty, \\ u(0, t) = u(L, t) = 0, \\ u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \end{cases}$$

in the following cases

- (a) $f(x) = 2 \sin(\frac{\pi x}{L}) + \sin(\frac{2\pi x}{L})$, $g(x) = 0$,
(b) $f(x) = 0$, $g(x) = \sin(\frac{n\pi x}{L})$, where n is a given integer.
(c) $f(x) = 2 \sin(\frac{\pi x}{L}) + \sin(\frac{2\pi x}{L})$, $g(x) = \sin(\frac{n\pi x}{L})$, where n is a given integer.
3. Use separation of variables to solve

$$u_{tt} = u_{xx} - u, \quad u(x, 0) = 0, \quad u_t(x, 0) = 1 + \cos^3 x,$$

on the interval $(0, \pi)$, with the homogeneous Dirichlet boundary conditions.

HOMEWORK POLICY

You are welcome to consult each other provided (1) you list all people and sources who aided you, or whom you aided and (2) you write-up the solutions independently, in your own language. If you seek help from other people, you should be seeking general advice, not specific solutions, and must disclose this help. This applies especially to internet fora such as **MathStackExchange**.

Similarly, if you consult books and papers outside your notes and the textbook, you should be looking for better understanding of or different points of view on the material, not solutions to the problems.