MATH 222 WRITTEN ASSIGNMENT 1

DUE TUESDAY MARCH 17

Note: Provide sufficient details and explanations in full sentences.

1. Give a convincing argument that the following inequalities are true
$$\int_0^n x^p \mathrm{d}x \leq 1 + 2^p + 3^p + \ldots + n^p \leq \int_0^{n+1} x^p \mathrm{d}x,$$
for any positive integer n and any real number $p \geq 0$. Then compute the limit
$$\lim_{n\to\infty} a_n, \qquad \text{where} \qquad a_n = \frac{1}{n^{p+1}} + \frac{2^p}{n^{p+1}} + \frac{3^p}{n^{p+1}} + \ldots + \frac{n^p}{n^{p+1}}.$$
2. Derive the Maclaurin series for the following functions.
   (a) $S(x) = \displaystyle\int_0^x \sin t^2 \,\mathrm{d}t$.
   (b) $C(x) = \displaystyle\int_0^x \frac{e^t + e^{-t} - 2}{t^2} \,\mathrm{d}t$.
3. Decide if the following series converge.
   (a) $\displaystyle\sum_{n=1}^\infty \log(n \sin \frac{1}{n})$.
   (b) $\displaystyle\sum_{n=2}^\infty \log \frac{n^2}{n^2 - 1}$.
4. Determine the convergence radius of the power series $\displaystyle\sum_{n=0}^\infty (\log n + a^n) x^n$, where $a \geq 0$.
5. For 3-dimensional vectors $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$, we define their *dot-* and *cross products* by
$$A \cdot B = a_1 b_1 + a_2 b_2 + a_3 b_3, \qquad \text{and}$$
$$A \times B = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1).$$
Show that
$$|A \times B|^2 = |A|^2 |B|^2 - |A \cdot B|^2,$$
where we recall that the *length* of a vector $X = (x_1, x_2, x_3)$ is given by
$$|X|^2 = x_1^2 + x_2^2 + x_3^2.$$

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6. Suppose that $r(t)$ is a positive function of a real parameter t, and consider the plane curve given by
$$X(t) = (r(t)\cos t, r(t)\sin t).$$
Derive formulas for the unit tangent $T(t)$, the principal unit normal $N(t)$, and the curvature $\kappa(t)$ for this curve. Note that your results will be expressions dependent on r, r', and r''. Apply the formulas to the special case
$$X(t) = (t^2\cos t, t^2\sin t), \qquad t \geq 0,$$
to compute its unit tangent, principal unit normal, and curvature.
7. Compute the curvature and torsion of the space curve
$$X(t) = (at, bt^2, ct^3), \qquad t \geq 0,$$
where a, b, and c are positive constants.
8. (Bonus question) Suggest a problem for the final exam. The subject may be anything up to and including space curves. Your submission must include the following components.
   - Problem statement.
   - Full solution.
   - Explanation as to why it is well suited to be on the final exam.
   - List of all people and sources who aided you.

   We will select one or two problems among the submissions, and will include them into the final exam. Apart from this, any reasonable submission will result in the *possibility to transfer the weight of your midterm exam to the final exam.* By a reasonable submission, we mean that
   - It is complete, with all the above-listed components.
   - It is on topic, and not completely trivial.
   - It is not taken from some source directly.
   - It can be a modification of a problem that occurs somewhere (In this case, the source must be cited).

Homework policy

You are welcome to consult each other provided (1) you list all people and sources who aided you, or whom you aided and (2) you write-up the solutions independently, in your own language. If you seek help from other people, you should be seeking general advice, not specific solutions, and must disclose this help. This applies especially to internet fora.

Similarly, if you consult books and other publications, you should be looking for better understanding of or different points of view on the material, not solutions to the problems.