# Diagonalization

If $A = PDP^{-1}$ with $D$ diagonal, $A$ is called diagonalizable.

$n \times n$ matrix $A$ is diagonalizable iff $A$ has $n$ linearly independent eigenvectors. If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are the eigenvectors, and $\lambda_1, \ldots, \lambda_n$ are the corresponding eigenvalues

$$A = PDP^{-1} \qquad \text{with} \qquad P = [\mathbf{v}_1 \ldots \mathbf{v}_n], \qquad D = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

Algorithm for diagonalizing $A$:

- Find the eigenvalues of $A$, i.e., solve the characteristic equation $\det(A - \lambda I) = 0$
- Find bases for the eigenspaces of $A$, i.e., basis for $\mathrm{Nul}\,(A - \lambda_k I)$ for each $\lambda_k$
- If there is enough linearly independent eigenvectors, construct $P$ and $D$
- Check: $A = PDP^{-1}$ or $AP = PD$

Some criteria:

- distinct eigenvalues, or symmetric matrix $\Rightarrow$ diagonalizable
- dimension of $\mathrm{Nul}\,(A - \lambda_k I)$ is less than the multiplicity of $\lambda_k \Rightarrow$ not diagonalizable