Lecture 20: Sections 5.1, 5.2

Eigenvectors and eigenvalues

Definition

Let A be a square matrix. If, for a nonzero vector $\mathbf{x}$,

$$A\mathbf{x} = \lambda\mathbf{x}$$

then $\mathbf{x}$ is called an eigenvector of A, and λ is called an eigenvalue of A.

α is an eigenvalue of $A \Leftrightarrow (A - \alpha I)\mathbf{x} = \mathbf{0}$ has a nontrivial solution $\Leftrightarrow$ $(A - \alpha I)$ is not invertible $\Leftrightarrow \det(A - \alpha I) = 0$

- $\det(A - \alpha I) = 0$ the characteristic equation
- $\det(A - \alpha I)$ the characteristic polynomial
- $\mathrm{Nul}\,(A - \lambda I)$ the eigenspace corresponding to the eigenvalue λ
- The eigenvalues of a triangular matrix are the entries on its main diagonal
- If $B = P^{-1}AP$ then A and B have the same characteristic polynomial and hence the same eigenvalues
- If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are eigenvectors that correspond to distinct eigenvalues of some matrix, then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly independent