Lecture 14 (Section 4.3)

Linear independence

Definition

A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in V is linearly independent if the vector equation

$$c_1\mathbf{v}_1 + \ldots + c_p\mathbf{v}_p = \mathbf{0}$$

has only the trivial solution.

- $\{\mathbf{v}\}$ is linearly dependent $\Leftrightarrow \mathbf{v} = \mathbf{0}$
- $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly dependent $\Leftrightarrow$ one of $\mathbf{v}_1$ and $\mathbf{v}_2$ is a multiple of the other
- $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent $\Leftrightarrow$ one of the vectors is a linear combination of the others

Basis

Definition

Let H be a subspace of V. A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a basis for H if

- $H = \text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$
- $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a linearly independent set

- If A is an invertible $n \times n$ matrix, then the columns of A form a basis for $\mathbb{R}^n$
- The pivot columns of $A = [\mathbf{a}_1 \ldots \mathbf{a}_p]$ form a basis for $\text{Span}\{\mathbf{a}_1 \ldots \mathbf{a}_p\}$ ($= \text{Col}\, A$)

Basis for $\text{Nul}\, A$:

- Write the solution of $A\mathbf{x} = \mathbf{0}$ as a linear combination of vectors where the weights are the free variables (parametric vector form)

$$\mathbf{x} = x_{i_1}\mathbf{u}_1 + \ldots + x_{i_q}\mathbf{u}_q$$

- The vectors $\{\mathbf{u}_1, \ldots, \mathbf{u}_q\}$ form a basis for $\text{Nul}\, A$