Lecture 9 (Section 2.1)

Some special matrices

Sums and scalar multiples of matrices

Matrix multiplication

Matrix transpose

Some special matrices

$k \times n$ matrix $A = [a_{ij}]$

- a_{ij} or $(A)_{ij}$ is the entry in the i-th row and j-th column of A
- Diagonal entries are a_{ii}, and they form the main diagonal of A
- If $a_{ij} = 0$ for all i and j, A is called the zero matrix of size $k \times n$
- If $k = n$, A is a square matrix
- If A is square and $a_{ij} = 0$ for $i \neq j$, A is a diagonal matrix
- If A is diagonal and $a_{ii} = 1$ for all i, it is the identity matrix of size n
- The identity matrix of size n is usually denoted by I_n, or just I

Sums and scalar multiples of matrices

Let $A = [a_{ij}]$ be a $k \times n$ matrix, and $B = [b_{ij}]$ be a $p \times q$ matrix

- $A = B$ if and only if $k = p$, $n = q$, and $a_{ij} = b_{ij}$ for all i and j
- If $k = p$ and $n = q$, then $A + B$ is the matrix with entries $a_{ij} + b_{ij}$, that is,

$$(A + B)_{ij} = (A)_{ij} + (B)_{ij}$$

- αA is the matrix with entries αa_{ij}, that is,

$$(\alpha A)_{ij} = \alpha (A)_{ij}$$

Properties of matrix sums and scalar multiples

Let A, B, C be matrices of the same size, and α and β be scalars.

- $A + B = B + A$ (commutative law)
- $(A + B) + C = A + (B + C)$ (associative law)
- $A + 0 = A$ (zero matrix)
- $\alpha(A + B) = \alpha A + \alpha B$ (distributive law)
- $(\alpha + \beta)A = \alpha A + \beta A$ (distributive law)
- $(\alpha\beta)A = \alpha(\beta A)$ (associative law)

Matrix multiplication

- A is $k \times n$
- B is $n \times p$ matrix with columns $\mathbf{b}_1, \ldots, \mathbf{b}_p$

then

- AB is $k \times p$ matrix with columns $A\mathbf{b}_1, \ldots, A\mathbf{b}_p$
- $AB = A[\mathbf{b}_1, \ldots, \mathbf{b}_p] = [A\mathbf{b}_1, \ldots, A\mathbf{b}_p]$
- $(AB)_{ij} = (A)_{i1}(B)_{1j} + (A)_{i2}(B)_{2j} + \ldots + (A)_{in}(B)_{nj}$

Properties of matrix multiplication

- $(AB)C = A(BC)$ (associative law)
- $A(B + C) = AB + AC$ (left distributive law)
- $(A + B)C = AC + BC$ (right distributive law)
- $\alpha(AB) = (\alpha A)B = A(\alpha B)$
- $I_k A = A = A I_n$ (identity for matrix multiplication)

Warnings

- $AB \neq BA$ for "most" of the matrices A and B
- $AB = AC$ for some $B \neq C$
- $AB = 0$ for some $A \neq 0$ and $B \neq 0$

Matrix transpose

Let A be $k \times n$. Then A^T is $n \times k$ matrix with entries $(A^T)_{ij} = (A)_{ji}$.

Properties:

- $(A^T)^T = A$
- $(A+B)^T = A^T + B^T$
- $(\alpha A)^T = \alpha A^T$
- $(AB)^T = B^T A^T$