

Lecture 5 (Section 1.6)

Leontief's Exchange Model

Balancing Chemical Equations

Leontief's Exchange Model

	Agriculture	Mining	Manufacturing
Agriculture	65%	20%	20%
Mining	5%	10%	30%
Manufacturing	30%	70%	50%

x_1, x_2, x_3 = prices of the total outputs of the sectors

$$\text{Agriculture's total expenses} = 0.65x_1 + 0.20x_2 + 0.20x_3$$

$$\text{Mining's total expenses} = 0.05x_1 + 0.10x_2 + 0.30x_3$$

$$\text{Manufacturing's total expenses} = 0.30x_1 + 0.70x_2 + 0.50x_3$$

For **equilibrium**, we need

$$\begin{cases} x_1 = 0.65x_1 + 0.20x_2 + 0.20x_3 \\ x_2 = 0.05x_1 + 0.10x_2 + 0.30x_3 \\ x_3 = 0.30x_1 + 0.70x_2 + 0.50x_3 \end{cases} \Leftrightarrow \begin{cases} -0.35x_1 + 0.2x_2 + 0.2x_3 = 0 \\ 0.05x_1 - 0.9x_2 + 0.3x_3 = 0 \\ 0.3x_1 + 0.7x_2 - 0.5x_3 = 0 \end{cases}$$

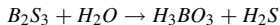
Leontief's Exchange Model

$$\begin{bmatrix} -0.35 & 0.2 & 0.2 & 0 \\ 0.05 & -0.9 & 0.3 & 0 \\ 0.3 & 0.7 & -0.5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -18 & 6 & 0 \\ -7 & 4 & 4 & 0 \\ 3 & 7 & -5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -18 & 6 & 0 \\ 0 & -122 & 46 & 0 \\ 0 & 61 & -23 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -18 & 6 & 0 \\ 0 & -122 & 46 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -18 & 6 & 0 \\ 0 & 1 & -46/122 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -0.7869 & 0 \\ 0 & 1 & -0.3770 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 = 0.7869x_3 \\ x_2 = 0.3770x_3 \\ x_3 \text{ is free} \end{cases} \quad \text{For instance, if } x_3 = 100, \quad \begin{cases} x_1 = 78.69 \\ x_2 = 37.70 \\ x_3 = 100 \end{cases}$$

Balancing Chemical Equations



$$(x_1)B_2S_3 + (x_2)H_2O = (x_3)H_3BO_3 + (x_4)H_2S$$

B: $2x_1 = x_3$ **S:** $3x_1 = x_4$ **H:** $2x_2 = 3x_3 + 2x_4$ **O:** $x_2 = 3x_3$

$$\begin{cases} 2x_1 - x_3 = 0 \\ 3x_1 - x_4 = 0 \\ 2x_2 - 3x_3 - 2x_4 = 0 \\ x_2 - 3x_3 = 0 \end{cases} \quad \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Balancing Chemical Equations

$$\sim \begin{bmatrix} 2 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 = \frac{1}{3}x_4 \\ x_2 = 2x_4 \\ x_3 = \frac{2}{3}x_4 \\ x_4 \text{ is free} \end{cases}$$

If $x_4 = 3$, then $x_1 = 1$, $x_2 = 6$, $x_3 = 2$

