$A\mathbf{x}=\mathbf{b}$, A is k by n matrix, $T(\mathbf{x})=A\mathbf{x}$

<table>
<tr><th></th><th>inconsistent</th><th colspan="2">consistent</th><th>consistent for all b in R^k</th></tr>
<tr><th></th><th></th><th>many solutions</th><th>unique solution</th><th></th></tr>
<tr><td>row reduction</td><td>last column of the augmented matrix [A b] is a pivot column</td><td>at least one non-pivot column in A</td><td>all columns of A are pivot columns</td><td>A has a pivot position in every row</td></tr>
<tr><td rowspan="2">linear combination and span</td><td rowspan="2">b is not in the span of the columns of A</td><td>columns of A are linearly dependent</td><td rowspan="2">columns of A are linearly independent</td><td>columns of A span R^k</td></tr>
<tr><td>at least one of the columns of A is a linear combination of the others</td><td>each b in R^k is a linear combination of the columns of A</td></tr>
<tr><td>homogeneous equation</td><td>-</td><td>Ax=0 has more than one solution</td><td>Ax=0 has only the trivial solution</td><td>-</td></tr>
<tr><td rowspan="2">mapping</td><td rowspan="2">b is not in the range of T</td><td>T is not one-to-one</td><td>T is one-to-one</td><td>T is onto</td></tr>
<tr><td>some b in R^k is the image of more than one vectors in R^n</td><td>each b in R^k is the image of at most one x in R^n</td><td>range of T is R^k</td></tr>
<tr><td rowspan="5">$k = n$</td><td colspan="2">A is not invertible</td><td colspan="2">A is invertible</td></tr>
<tr><td colspan="2">not $A \sim I_n$</td><td colspan="2">$A \sim I_n$</td></tr>
<tr><td colspan="2">there is no C such that $CA=I_n$</td><td colspan="2">there is C such that $CA=I_n$</td></tr>
<tr><td colspan="2">there is no D such that $AD=I_n$</td><td colspan="2">there is D such that $AD=I_n$</td></tr>
<tr><td colspan="2">A^T is not invertible</td><td colspan="2">A^T is invertible</td></tr>
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