$A\mathbf{x}=\mathbf{b}$, $A=[\mathbf{a}_1,\dots,\mathbf{a}_n]$

<table>
<tr><td rowspan="2"></td><td rowspan="2">inconsistent</td><td colspan="2">consistent</td></tr>
<tr><td>unique solution</td><td>many solutions</td></tr>
<tr><td>row reduction algorithm</td><td>last column of the augmented matrix [A b] is a pivot column</td><td>all columns of A are pivot columns</td><td>at least one non-pivot column in A</td></tr>
<tr><td rowspan="2">vector language (column picture)</td><td rowspan="2">b is not in the span of $\mathbf{a}_1,\dots,\mathbf{a}_n$</td><td rowspan="2">$\mathbf{a}_1,\dots,\mathbf{a}_n$ are linearly independent</td><td>$\mathbf{a}_1,\dots,\mathbf{a}_n$ are linearly dependent</td></tr>
<tr><td>at least one of the vectors $\mathbf{a}_1,\dots,\mathbf{a}_n$ is a linear combination of the others</td></tr>
<tr><td rowspan="2">matrix language</td><td rowspan="2">b is not in the span of the columns of A</td><td>columns of A are linearly independent</td><td rowspan="2">columns of A are linearly dependent</td></tr>
<tr><td>$A\mathbf{x}=\mathbf{0}$ has only the trivial solution</td></tr>
</table>

- Row reduction algorithm, related terminology
- Linear combination, span, linear independence
- Matrix-vector multiplication
- Is $A\mathbf{x}=\mathbf{b}$ consistent for every $\mathbf{b}$ in R^k?
- If $\mathbf{b}=\mathbf{0}$, nontrivial solution <=> at least one *free* variable (or *non-pivot* column)

$A\mathbf{p}=\mathbf{b}$, $A\mathbf{v}=\mathbf{0}$ => $A(\mathbf{p}+\mathbf{v})=\mathbf{b}$

$A\mathbf{p}=\mathbf{b}$, $A\mathbf{x}=\mathbf{b}$ => $A(\mathbf{x}-\mathbf{p})=\mathbf{0}$