MATH 20F WINTER 2007 PRACTICE MIDTERM II

FEBRUARY 28

GUIDELINES:

- *Please put your* **name**, **ID number**, **TA's name**, *and* **section time** *on your blue book or exam sheet.*
- *No books, notes, or calculators are allowed.*
- *Write your solutions clearly and give explanations for your work. Answers without justifications will not be given credit.*
- *If any question is not clear, ask for clarification.*

PROBLEMS:

1. [*10pts*] Let A be a 3×3 matrix such that $\det A = 4$, and let B be given by

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}.$$

a). [*5pts*] Calculate $\det(A^{-1}B)$.
b). [*5pts*] Write B in LU form, that is, find an upper triangular matrix U and a unit lower triangular matrix L such that $B = LU$.

2. [*10pts*] Determine if each of the following set is a vector space. If so, find a basis for it and calculate its dimension. (*Hint*: If a subset of a vector space is itself a vector space, it is called a subspace.)

a). [*5pts*]

$$S = \{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n : x_1 + \ldots + x_n = 0 \}.$$

b). [*5pts*]

$$H = \{ \begin{bmatrix} \alpha \\ \beta \\ \alpha \\ \beta \end{bmatrix} \in \mathbb{R}^4 : \alpha, \beta \in \mathbb{R} \}.$$

(SEE OTHER SIDE)

3. [*20pts*] Let the following vectors be given:

$$\mathbf{u}_1 = \begin{bmatrix} 2 \\ -4 \\ 0 \end{bmatrix}, \qquad \mathbf{u}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \qquad \mathbf{u}_3 = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix},$$

and

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}.$$

a). [*5pts*] Prove that $\mathcal{U} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a basis for $\mathbb{R}^3$.

b). [*5pts*] Find the change-of-coordinate matrix from $\mathcal{U}$ to $\mathcal{V} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, that is, find the matrix P such that $[\mathbf{x}]_\mathcal{V} = P[\mathbf{x}]_\mathcal{U}$ for all $\mathbf{x} \in \mathbb{R}^3$. Is $\mathcal{V}$ a basis for $\mathbb{R}^3$?

c). [*5pts*] If the coordinate vector of $\mathbf{x} \in \mathbb{R}^3$ relative to the basis $\mathcal{U}$ is given by

$$[\mathbf{x}]_\mathcal{U} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix},$$

find $[\mathbf{x}]_\mathcal{V}$, that is, the coordinate vector of $\mathbf{x}$ relative to the basis $\mathcal{V}$.

d). [*5pts*] Find the change-of-coordinate matrix from $\mathcal{V}$ to $\mathcal{U}$, that is, find the matrix Q such that $[\mathbf{x}]_\mathcal{U} = Q[\mathbf{x}]_\mathcal{V}$ for all $\mathbf{x} \in \mathbb{R}^3$. (*Hint*: How is Q related to P?)

4. [*10pts*] Mark each statement TRUE or FALSE. Briefly justify each answer.

a). In order for a matrix B to be the inverse of A, both equations $AB = I$ and $BA = I$ must be true.

b). If A is an invertible $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent and has a unique solution for each $\mathbf{b}$ in $\mathbb{R}^n$.

c). $\det A = -\det A^T$.

d). A basis is a spanning set that is as large as possible.

e). A basis is a linearly independent set that is as large as possible.