MATH 20F WINTER 2007 MIDTERM EXAM II

FEBRUARY 28

GUIDELINES:

- *Please put your* **name**, **ID number**, **TA's name**, *and* **section time** *on your blue book or exam sheet.*
- *No books, notes, or calculators are allowed.*
- *Write your solutions clearly and give explanations for your work. Answers without justifications will not be given credit.*
- *If any question is not clear, ask for clarification.*

PROBLEMS:

1. [*10pts*] Let B be a 3×3 matrix such that $\det B = 2$, and let A be given by

$$A = \begin{bmatrix} 1 & 3 & 9 \\ 1 & 8 & 64 \\ 1 & 13 & 169 \end{bmatrix}.$$

   a). [*5pts*] Calculate $\det(BAB)$.

   b). [*5pts*] Write A in LU form, that is, find an upper triangular matrix U and a unit lower triangular matrix L such that $A = LU$.

2. [*10pts*] Let the matrix A and the vectors $\mathbf{u} \in \mathbb{R}^4$ and $\mathbf{w} \in \mathbb{R}^3$ be given by

$$A = \begin{bmatrix} 3 & -2 & 4 & 4 \\ -2 & 6 & 2 & 0 \\ 4 & 2 & 3 & 9 \end{bmatrix}, \qquad \mathbf{u} = \begin{bmatrix} 14 \\ 10 \\ -1 \\ -14 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} 23 \\ 43 \\ 0 \end{bmatrix}.$$

   a). [*5pts*] Find a *basis* for and the *dimension* of $\operatorname{Nul} A$. Is $\mathbf{u}$ in $\operatorname{Nul} A$? (*Hint on row reducing* A: Scale the second row by factor $\frac{1}{2}$ and interchange the first two rows. Then do not use scaling until the last moment. This will save some arithmetics on fractions.)

   b). [*5pts*] Find a *basis* for and the *dimension* of $\operatorname{Col} A$. Is $\mathbf{w}$ in $\operatorname{Col} A$?

(SEE OTHER SIDE)

3. [*10pts*] Let H be a vector space and let $\mathcal{V} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for H. Let the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ be given by

$$\mathbf{u}_1 = \mathbf{v}_1 - 2\mathbf{v}_3, \qquad \mathbf{u}_2 = -3\mathbf{v}_1 + \mathbf{v}_2 + 4\mathbf{v}_3, \qquad \mathbf{u}_3 = 2\mathbf{v}_1 - 3\mathbf{v}_2 + 4\mathbf{v}_3.$$

   a). Is $\mathcal{U} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ a basis for H? If so, find the *change-of-coordinate matrix* from $\mathcal{V}$ to $\mathcal{U}$, that is, find the matrix Q such that $[\mathbf{x}]_{\mathcal{U}} = Q[\mathbf{x}]_{\mathcal{V}}$ for all $\mathbf{x} \in H$. Here $[\mathbf{x}]_{\mathcal{V}}$ and $[\mathbf{x}]_{\mathcal{U}}$ denote the *coordinate vectors* of $\mathbf{x}$ relative to the bases $\mathcal{V}$ and $\mathcal{U}$, respectively.
   b). If $\mathbf{x} = 4\mathbf{v}_1 + \mathbf{v}_2 - 3\mathbf{v}_3$, find $[\mathbf{x}]_{\mathcal{U}}$.

4. [*10pts*] Mark each statement TRUE or FALSE. Briefly justify each answer.
   a). If A and B are invertible $n \times n$ matrices, then $A^{-1}B^{-1}$ is the inverse of AB.
   b). If A is invertible, then elementary row operations that reduce A to I also reduce I to A^{-1}.
   c). The determinant of a matrix in echelon form is the product of its pivot entries.
   d). The number of pivot columns in A is the dimension of $\operatorname{Nul} A$.
   e). If V is a vector space of dimension k, and $\mathbf{x}$ is in V, then the coordinate vector of $\mathbf{x}$ relative to any basis for V is in $\mathbb{R}^k$.