MATH 557 - ASSIGNMENT 3

To be handed in not later than 5pm, 20th March 2008.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Let $X_1, \ldots, X_n$ be a random sample from the $Beta(1, \theta)$ probability model, for parameter $\theta > 0$.

(a) Find the Uniformly Most Powerful (UMP) level α test (that is, the form of the test statistic and rejection region) of hypotheses

$$\begin{aligned} H_0 &: \theta = 1 \\ H_1 &: \theta > 1 \end{aligned}$$

4 MARKS

(b) Find the Likelihood Ratio Test (LRT) for testing

$$\begin{aligned} H_0 &: \theta = 1 \\ H_1 &: \theta \neq 1 \end{aligned}$$

that has level α.

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2 Find the UMP level α test (that is, the form of the test statistic and rejection region) for hypotheses

$$\begin{aligned} H_0 &: \theta \leq \theta_0 \\ H_1 &: \theta > \theta_0 \end{aligned}$$

where $\theta > 0$, and θ_0 is a fixed positive constant, based on a random sample of size n from the following probability models:

(a) Exponential$(1/\theta)$:

$$f_{X|\theta}(x|\theta) = \frac{1}{\theta} e^{-x/\theta} \qquad x > 0$$

(b) Normal$(1, \theta)$:

$$f_{X|\theta}(x|\theta) = \left(\frac{1}{2\pi\theta}\right)^{1/2} \exp\left\{-\frac{(x-1)^2}{2\theta}\right\} \qquad -\infty < x < \infty$$

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3 Suppose that $X_1, \ldots, X_n \sim$ Poisson(θ) for $\theta > 0$ is a random sample. Construct a (randomized) test of the hypotheses

$$\begin{aligned} H_0 &: \theta \leq 2 \\ H_1 &: \theta > 2 \end{aligned}$$

that is a UMP level $\alpha = 0.05$ test, that is, where

$$\alpha = \Pr[T(\underset{\sim}{X}) \in \mathcal{R}_T | \theta]$$

for suitably chosen test statistic $T(\underset{\sim}{X})$ and rejection region $\mathcal{R}_T$. Report the outcome of the test for the data set

$$2 \quad 3 \quad 5 \quad 1 \quad 5 \quad 2.$$

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