MATH 557 - ASSIGNMENT 2

To be handed in not later than 5pm, 14th February 2008.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Suppose that $X_1, \ldots, X_n$ are a random sample from the pdf

$$f_{X|\theta}(x|\theta) = \frac{1}{\theta}\exp\{-(x-\theta)/\theta\} \qquad x > \theta$$

and zero otherwise, for some $\theta > 0$.

(a) Find a (possibly multivariate) minimal sufficient statistic, $\underset{\sim}{T}(\underset{\sim}{X})$, for θ.

4 MARKS

(b) Is $\underset{\sim}{T}(\underset{\sim}{X})$ a complete statistic ? Justify your answer.

4 MARKS

2 Suppose that $X_1, \ldots, X_n$ are a random sample from the pdf

$$f_{X|\theta}(x|\theta) = \frac{1+\theta x}{2} \qquad -1 < x < 1 \tag{1}$$

and zero otherwise, for some θ where $0 < \theta < 1$.

(a) An estimator $\theta_n(\underset{\sim}{X})$ is **consistent** for θ if $\theta_n(\underset{\sim}{X})$ converges to θ,

$$\theta_n(\underset{\sim}{X}) \longrightarrow \theta$$

as $n \longrightarrow \infty$, where convergence is *in probability*, *almost surely*, or in *mean-square* (rth mean for $r = 2$).

Find a consistent estimator for θ, to be denoted $\widetilde{\theta}_n(\underset{\sim}{X})$, in the model in equation (1).

2 MARKS

(b) Find the **asymptotic variance** of $\widetilde{\theta}_n(\underset{\sim}{X})$ as $n \longrightarrow \infty$, defined here as the variance of the limiting distribution of the random variable

$$n^{\alpha}(\widetilde{\theta}_n(\underset{\sim}{X}) - \theta)$$

where $\alpha > 0$ is an appropriately chosen constant.

2 MARKS

3 Suppose that $X_1, \ldots, X_n$ are a random sample from a $Poisson(\theta)$ distribution, where $\theta > 0$. Let $\phi = \Pr[X_i = 0] = e^{-\theta}$, and consider the two estimators of ϕ given by

$$\widetilde{\phi}_{n1}(\underset{\sim}{X}) \equiv T_n = \frac{1}{n}\sum_{i=1}^{n} I_{\{0\}}(X_i) \qquad\qquad \widetilde{\phi}_{n2}(\underset{\sim}{X}) \equiv M_n = e^{-\overline{X}_n}$$

Using the Central Limit Theorem, and the Delta Method, find the **asymptotic efficiency** of $\widetilde{\phi}_{n1}(\underset{\sim}{X})$ relative to $\widetilde{\phi}_{n2}(\underset{\sim}{X})$, defined here as the ratio of the asymptotic variances of the two estimators.

8 MARKS